4.6 Modeling with Exponential and Logarithmic Functions

Exponential Growth and Decay Models: predict any quantity that grows or decays by a fixed percent at regular intervals

Mathematical Model for Exponential Growth or Decay

$$f(t) = y = y_0 e^{kt}$$

$f(t) = y$: the amount of the substance at time t

y_0 : the original amount at $t = 0$

k : the growth or decay rate (decimal)

$k > 0$: law of uninhibited growth

$k < 0$: law of decay

t : time (years, days, hours, minutes, etc.)

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Continuous Compounding: $A = Pe^{rt}$

Ex. (#16) Suppose that \$50,000 from a retirement account is invested in a large cap stock fund. After 20 yr, the value is \$194,809.67.

a.) Use the model $A = Pe^{rt}$ to determine the average rate of return under continuous compounding.

b.) How long will it take the investment to reach one quarter million dollars? Round to 1 decimal place.

Ex. On January 1, 2000, the population of Texas was approximately 21 million. On January 1, 2010, the population was 25.2 million.

a.) Write a function of the form $P(t) = P_0 e^{kt}$ to represent the population $P(t)$ of Texas t years after January 1, 2000. Round k to 5 decimal places.

b.) Use the function in part (a) to predict the population on January 1, 2020. Round to 1 decimal place.

c.) Use the function in part (a) to determine the year during which the population of Texas will reach 40 million if this trend continues.

The **half-life** of a substance is the time required for half of original substance to disintegrate.

Ex. The half-life of the radioactive element plutonium-239 is 25,000 years. If 16 grams of plutonium-239 are initially present, how many grams are present after 25,000 years? 50,000 years? 75,000 years? 100,000 years? 125,000 years?

Ex. The half-life of aspirin in your bloodstream is 12 hours.

a.) Find the exponential decay model for aspirin in bloodstream.

b.) How long will it take for the aspirin to decay to 70% of the original dosage? (Round the answer to one decimal place.)

The percentage of **carbon-14** remaining in the fossil or artifact is used to determine the age of fossils and artifacts.

Exponential Decay Model for Carbon-14: $Q(t) = Q_0 e^{-0.000121t}$

Ex. Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected amount of carbon-14 found in living person. In 1989, how old were the skeletons?